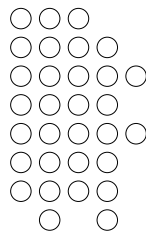


計算組合數 Comb(n,k)

丁培毅



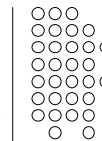
計算 C_k^n and P_k^n

• 版本一 long int

```

01 #include <stdio.h> 17 int find_nCk(int n, int k) {
02 #include <stdlib.h> 18 return factorial(n) / (factorial(k)*factorial(n-k));
03 int factorial(int); 19 }
04 int find_nCk(int, int);
05 int find_nPk(int, int);
06 int main(void) {
07     int n, k, nCk, nPk;
08     printf("Enter the value of n and k\n");
09     scanf("%d%d",&n,&k);
10     nCk = find_nCk(n, k);
11     nPk = find_nPk(n, k);
12     printf("%dC%d = %d\n", n, k, nCk);
13     printf("%dP%d = %d\n", n, k, nPk);
14     system("pause");
15     return 0;
16 }
    
```

$$\frac{n!}{k!(n-k)!}$$



```

20 int find_nPk(int n, int k) {
21     return factorial(n)/factorial(n-k);
22 }
    
```

```

23 int factorial(int n) {
24     int i, result=1;
25     for (i=1; i<=n; i++)
26         result = result*i;
27     return result;
28 }
    
```

$$P_3^{15} = 15 \cdot 14 \cdot 13 = 2730$$

$$15! = 1307674368000$$

$$2^{31}-1 = 2147473647$$

這樣子寫理論上是對的，
實際上卻有很嚴重的問題!!

計算 C_k^n and P_k^n

• 版本二 long long int

```

01 #include <stdio.h> 17 int find_nCk(int n, int k) {
02 #include <stdlib.h> 18 return factorial(n)/(factorial(k)*factorial(n-k));
03 long long int factorial(int); 19 }
04 int find_nCk(int, int);
05 int find_nPk(int, int);
06 int main(void) {
07     int n, k, nCk, nPk;
08     printf("Enter the value of n and k\n");
09     scanf("%d%d",&n,&k);
10     nCk = find_nCk(n, k);
11     nPk = find_nPk(n, k);
12     printf("%dC%d = %d\n", n, k, nCk);
13     printf("%dP%d = %d\n", n, k, nPk);
14     system("pause");
15     return 0;
16 }
    
```

```

23 long long int factorial(int n) {
24     int i; long long int result=1;
25     for (i=1; i<=n; i++)
26         result = result*i;
27     return result;
28 }
    
```

$$P_3^{21} = 21 \cdot 20 \cdot 19 = 7980$$

$$21! = 51090942171709440000$$

$$2^{63}-1 = 9223372036854775807$$

好一些，但是仍然
有很大的限制

計算 C_k^n and P_k^n

• 版本三 skip unnecessary factorial n!, (n-k)!

```

long long int find_nPk(int n, int k) {
    int i;
    long long int result=1;
    for (i=0; i<k; i++)
        result = result * (n-i);
    return result;
}
    
```

$$P_k^n = n(n-1) \dots (n-k+1)$$

```

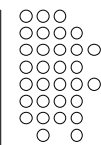
int find_nPk(int n, int k) {
    int i, result=1;
    for (i=0; i<k; i++)
        result = result * (n-i);
    return result;
}
    
```

Ex. 30P6 = 427518000
 30P7 = 1670497408
 30P8 = -233265280
 100P5 = 444567808
 100P6 = -715731200

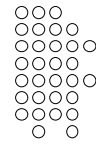
Ex. 30P13 = 745747076954880000
 30P14 = -5769043765476591616
 100P10 = 7475418734400817152
 100P11 = 8704899442529685504
 100P12 = -27200710659158016

```

printf("result = %I64d\n", result);
printf("result = %lld\n", result);
    
```



計算 C_k^n and P_k^n



● 版本三 (cont'd)

```
long long find_nCk(int n, int k) {
    int i;
    long long int numerator=1, denominator=1;
    for (i=0; i<k; i++) {
        numerator *= n-i;
        denominator *= k-i;
    }
    return numerator/denominator;
}
30C14 = -66175233
100C10 = 2060025003968
```

$$C_k^n = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1}$$

```
long long find_nCk(int n, int k) {
    int i;
    long long numerator=1;
    long long denominator=1;
    for (i=0; i<k; i++) {
        numerator *= n-i;
        if (numerator % (k-i) == 0)
            numerator /= k-i;
        else
            denominator *= k-i;
    }
    return numerator/denominator;
}
100C15 = 3327654233778599
```

```
long long int find_nCk(int n, int k) {
    int i;
    double result=1.0;
    for (i=0; i<k; i++)
        result *= ((double)(n-i))/(k-i);
    return (long long int) (result+0.5);
}
100C18 = -9223372036854775808
```

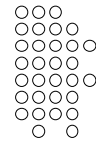
程式該怎樣寫才能計算出 C_{33}^{67}



```
• Comb(66,33) = 7,219,428,434,016,265,740
2^63-1 = 9,223,372,036,854,775,807
Comb(67,33) = 14,226,520,737,620,288,370
2^64-1 = 18,446,744,073,709,551,616
```

- 由前面版本三的程式你可以看到直接讓分子乘分子、分母乘分母的話很快就爆表了，第二個程式只是把正下方可以整除的分母除掉就可以看到改善了（沒有試看看其它的分母，也沒有嘗試把公因數除掉）
- 因為 $Comb(n,k)$ 一定是整數，所有的分母拆解成質因數乘積一定可以和分子的質因數乘積消掉，所以基本的方法就是記住每一個分子和分母，每一個分母都和所有的分子找公因數，除掉公因數

計算 C_{33}^{67}



● 需要計算最大公因數的程式

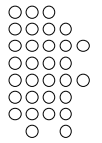
```
01 int gcd(int p, int q) { // assume p >= q
02     if (p % q == 0)
03         return q;
04     else
05         return gcd(q, p % q);
06 }
```

● 動手計算看看

$$C_5^{10} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$$

- 要像上面這樣子算的話，程式需要記住所有部份消去的分子，分母一定都會被消完

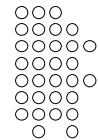
記憶體是很好用的東西



- 可以設計一個陣列紀錄所有的分子 10, 9, 8, 7, 6, 然後每一個分母denum和所有的分子num[j]一個一個找最大公因數d來消去，直到每一個分母都完全消去成為1（這個條件不需要檢查，只需要把所有分子試完）

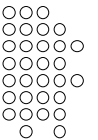
```
int n=10, k=5, i, j, d, denum, nCk;
int num[67] = {10, 9, 8, 7, 6}; // 先暴力放進去
for (i=k; i>1; i--)
    for (denum=i, j=0; denum>1 && j<k; j++)
        if ((num[j]>1) && ((d=gcd(denum, num[j]))>1))
            denum/=d, num[j]/=d; // 消去 d
for (nCk=1, i=0; i<k; i++) nCk *= num[i];
printf("nCk = %d\n", nCk);
```

如果不允許使用額外的記憶體



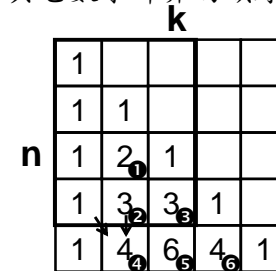
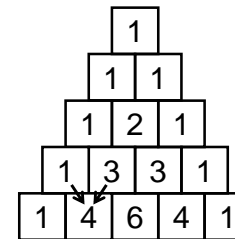
- 版本三的第二個程式就是沒有用陣列來紀錄分子，也沒有考慮分母是可以拆開成為質因數乘積，在不同的分子裡找到可以消去的質因數
- 如果不使用額外記憶體，要稍微改進一下的話，第一就是 numerator 不需要整除 k-i，只需要把最大公因數 $d = \text{gcd}(\text{numerator} * (n-i), \text{denominator} * (k-i))$ 消去就可以，numerator 和 denominator 兩個變數裡累積沒有完全消去的因數，不過這樣子計算時，還是有可能在計算中間數值 $\text{numerator} * (n-i)$ 或是 $\text{denominator} * (k-i)$ 時比 $\text{Comb}(n,k)$ 提早產生溢位

如果允許使用更多的記憶體



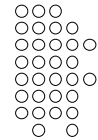
- 我們可以運用遞推公式來計算 $\text{Comb}(67,33)$

$$\text{Comb}(n,k) = \text{Comb}(n-1,k-1) + \text{Comb}(n-1,k)$$
- 這種計算需要清楚地掌握資料陣列的配置方法以及資料計算的順序，下圖左是一般的帕斯卡三角形
- 下圖右我們用二維的陣列來放 $\text{Comb}(n,k)$ ，n 是橫列，k 是直行， $k=0$ 和 $n=k$ 時固定都是 1，其它數字計算的順序標示如下



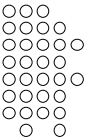
如果只需要計算 $\text{Comb}(n^*,k^*)$ 只須計算小於等於 n^*, k^* 的就足夠了

帕斯卡三角形



- 假設我們定義 `long long comb[67][67];` 的二維陣列
 1. 先把固定的數值填好: `comb[*][0], comb[i][i]`
`for (i=0; i<=n; i++) comb[i][0] = comb[i][i] = 1;`
 2. 由第三列開始一列一列往下填
`for (i=2; i<=n; i++)`
`for (j=1; j<i && j<=k; j++)`
`comb[i][j] = comb[i-1][j-1] + comb[i-1][j];`
- 如果使用的記憶體數量是很重要的考量，這個程式也可以只使用兩列的記憶體，`long long comb[2][67];` 由前一列的組合數算本列的組合數，兩列的記憶體輪流切換
- 請注意程式多使用記憶體以後演算法變得非常簡潔

偵測「溢位」- 純軟體

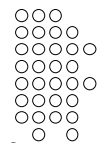


```
#define INT64_MIN_VALUE (0x8000000000000000LL)
#define INT64_MAX_VALUE (0x7FFFFFFFFFFFFFFFLL)
int isMultOverflow(long long a, long long b) {
    long long ap, bp, cp, max=INT64_MAX_VALUE;
    if (a==INT64_MIN_VALUE) return b!=1;
    if (b==INT64_MIN_VALUE) return a!=1;
    ap = a >= 0 ? a : -a;
    bp = b >= 0 ? b : -b;
    cp = max / bp;
    return (ap > cp);
}
```

計算每一個乘法之前，先判斷一下是否會發生溢位，不會的話才乘起來

等於是花兩倍的時間像是計算 $\text{Comb}(500, n)$ 真的不知道 64 位元可以算到多大的 n 值，只好邊算邊測試

偵測「溢位」- 硬體狀態旗標



- 主要運用CPU的指令 **JNO** (jump no overflow)

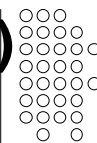
```
gcc/g++
int isOverflow()
{
    __asm__ ("movl $0, %eax;");
    __asm__ ("jno end;");
    __asm__ ("movl $1, %eax;");
    __asm__ ("end;");
}
```

如果偵測位置離開
運算敘述太遠就會
沒有效果

```
Visual C/C++
int isOverflow()
{
    __asm mov  eax, 0
    __asm jno  end
    __asm mov  eax, 1
    __asm end;
}
```

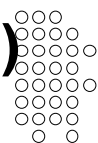
vc2010 把64位元乘法
用一個函式來完成, 這
種偵測就完全無效

C++ Exception Handling (1/3)



- There are three exception handling mechanisms you can use with C++ on MS Windows:
 - **C++ exceptions**, implemented by the compiler (**try/catch**), cl /EHsc
 - **Structured Exception Handling (SEH)**, provided by Windows (**__try / __except**), cl /EHa
 - **MFC exception macros** (TRY, CATCH - built on top of SEH / C++ exceptions - see also TheUndeadFish's comment)
- C++ exceptions guarantee **automatic cleanup** during stack unwinding (i.e. dtors of local objects), other mechanisms don't.
- C++ exceptions only occur when they are **explicitly thrown**. Structured Exceptions may occur for any operations (especially **divide by zero**, H/W, and invalid memory access, S/W).
- MFC introduces the macros to support exception handling even if compilers didn't implement them.

C++ Exception Handling (2/3)



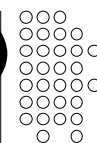
```
for (y=3; y>=0; y--)
try {
    int x=1, *z=0;
    if (y==3) throw exception("explicit");
    else if (y==2) throw 'a';
    else if (y==1) *z = 10;
    else
        x = x/y;
    cout << "end\n";
}
catch (exception) {
    cout << "exception caught\n";
}
catch (char e) {
    cout << "char exception caught\n";
}
catch (...) {
    cout << "... caught\n";
}
```

```
cl /EHsc
testCException.cpp
exception caught
char exception caught
```



```
cl /EHa testCException.cpp
exception caught
char exception caught
... caught
... caught
```

C++ Exception Handling (3/3)

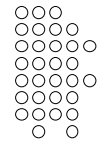


```
for (y=3; y>=0; y--)
__try {
    int x=1, *z=0;
    if (y==3)
        RaiseException(15,0,0,NULL);
    else if (y==2) throw 'a';
    else if (y==1) *z = 10;
    else
        x = x/y;
    cout << "end\n";bo
}
__except (FilterFunction(GetExceptionCode())) {
    cout << GetExceptionCode() << " exception caught\n";
}
```

```
DWORD FilterFunction(unsigned int code) {
    if (code == 15)
        return EXCEPTION_CONTINUE_SEARCH;
    else
        return EXCEPTION_EXECUTE_HANDLER;
}
```

Something like rethrow
Both cl /EHsc or cl /EHa work.

更大的組合數 $\text{Comb}(500,250)$



- 需要使用大整數類別來運算
例如 Crypto++, GMP, Java

```
Integer result;  
result = 1;  
for (i=0; i<n; i++) {  
    d = gcd(m-i, i+1);  
    num = (m-i) / d;  
    denum = (i+1) / d;  
    result /= denum;  
    result *= num;  
}
```

此程式輸入 $m \geq n \geq 0$, 輸出組合數 $\text{comb}(m, n)$:

請輸入 m: 500

請輸入 n: 250

$\text{comb}(500,250)=$

11674431578827768292093473476217661965923008118031144612410028495
781111267360847371566641777552160537681086590270998958016003746822
6393900042796872256.