電視有獎猜題

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Television quiz shows

- ABC 1999: Who wants to be a millionaire
- NBC 1964 Merf Griffin: Jeopardy
- NBC 1975 Merf Griffin: Wheel of Fortune (word puzzle like hangman)

知識王 app

Slumdog Millionaire

- The player starts with a prize of \$1, and is asked a sequence of n questions.
  - For each question, he may
    - quit and keep his prize
    - answer the question. If wrong, he quits with nothing. If correct, the prize is doubled, and he continues with the next question.
  - After the last question, he quits with his prize.
- The player wants to maximize his expected prize. Once each question is asked, the player is able to assess the probability p that he will be able to answer it. For each question, we assume that p is a random variable uniformly distributed over the range [t, 1], for $0 \le t \le 1$.

Sample Input/Output

- Input
  - Input is a number of lines, each with two numbers: an integer 1 <= n <= 30, and a real 0 <= t <= 1. Input is terminated by a line containing 0 0. This line should not be processed.
- Output

  For each input n and t, print the player's expected prize, if he plays the best strategy. Output should be rounded to three fractional digits.

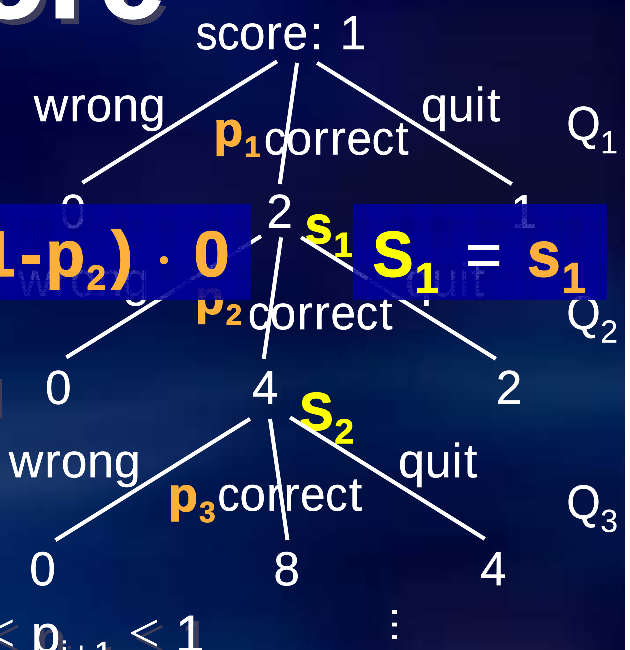
- Sample Input

  1 0.5
  1 0.3
  2 0.6
  24 0.25
  0 0

- Sample Output

  1.500
  1.357
  2.560
  230.138

Expected Score



$S_1 = p_2 \cdot S_2 + (1\text{-}p_2) \cdot 0$ $S_1 = s_1$

- **Player's best strategy**: **quit** if the score of not answering is larger than the expected score of answering; **answer** otherwise
- Let S_i be the expected score after Q_i is answered
  let s_i be the current score after Q_i is answered

  expected score S_i is s_i if quit answering
  expected score S_i is $p_{i+1} \cdot S_{i+1} + (1\text{-}p_{i+1}) \cdot 0$, $t \leq p_{i+1} \leq 1$
- The best strategy depends on the actual probability p_{i+1}, let $h = s_i / S_{i+1}$

  For $h \geq t$, the best strategy is **quit** if $t \leq p_{i+1} < h = s_i / S_{i+1}$ $(p_{i+1} S_{i+1} < s_i)$
  the best strategy is **answer** if $h \leq p_{i+1} \leq 1$ $(p_{i+1} S_{i+1} \geq s_i)$

the expected score after Q_i is answered
$$S_i = \int_t^h \frac{s_i}{1-t} dp_{i+1} + \int_h^1 \frac{p_{i+1} S_{i+1}}{1-t} dp_{i+1} = \frac{h-t}{(1-t)} s_i + \frac{1-h^2}{2(1-t)} S_{i+1}$$

For $h < t \leq p_{i+1}$, the best strategy is **answer** $S_i = \int_t^1 \frac{p_{i+1} S_{i+1}}{1-t} dp_{i+1} = \frac{1-t^2}{2(1-t)} S_{i+1}$

Recursive Version

```c
#include <stdio.h>
#include <stdlib.h>
double expectedScore(double s_i, int k, double t) {
    if (k == 0)
        return s_i;
    else {
        double S_ip1 = expectedScore(2*s_i, k-1, t);
        double h = s_i/S_ip1;
        if (t > h)
            h = t;
        return (s_i*(h-t) + S_ip1*(1-h*h)/2) / (1-t);
    }
}
int main(void) {
    int n;
    double t;
    while (scanf("%d %lf", &n, &t) == 2 && n != 0)
        printf("%.3lf\n", expectedScore(1, n, t));
    system("pause");
    return 0;
}
```

Iterative Version

$$S_i = \int_t^h \frac{s_i}{1-t} dp_{i+1} + \int_h^1 \frac{p_{i+1} S_{i+1}}{1-t} dp_{i+1} = \frac{h-t}{(1-t)} s_i + \frac{1-h^2}{2(1-t)} S_{i+1}$$

```
double expectedScore(int n, double t) {
    double s_i, S_ip1, h;
    int k;
    s_i = pow(2, n-1); S_ip1 = s_i*2;
    for (k=n; k>0; k--) { /* k-th question */
        h = s_i / S_ip1;
        if (t > h) h = t;
        S_ip1 = (s_i*(h-t) +
                    S_ip1*(1-h*h)/2) / (1-t);
        s_i /= 2;
    }
    return S_ip1;
}
```