More Examples w/o Using Arrays

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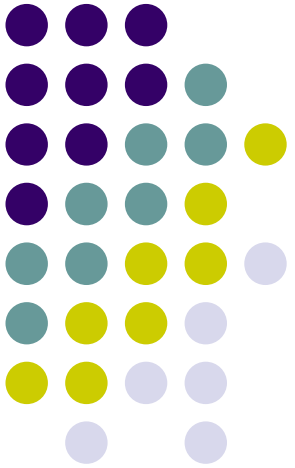
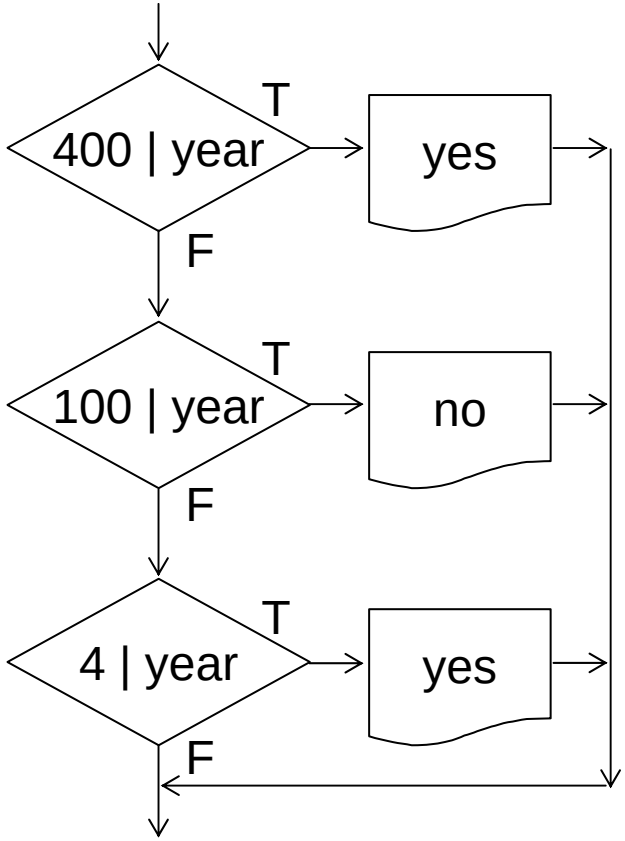
NTU math C prog

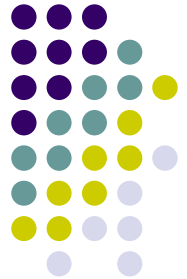


Check Leap Years

```
#include <stdio.h>
#include <stdlib.h>
int main(void) {
    int year;
    printf("Enter a year to check if it is a leap year\n");
    scanf("%d", &year);
    if ( year%400 == 0)
        printf("%d is a leap year.\n", year);
    else if ( year%100 == 0)
        printf("%d is not a leap year.\n", year);
    else if ( year%4 == 0 )
        printf("%d is a leap year.\n", year);
    else
        printf("%d is not a leap year.\n", year);
    system("pause");
    return 0;
}
```



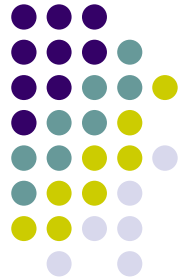
Check Vowels



- **Version 1**

```
#include <stdio.h>
#include <stdlib.h>
int main(void) {
    char ch;
    printf("Enter a character: ");
    scanf("%c", &ch);
    if (ch == 'a' || ch == 'A' || ch == 'e' || ch == 'E' ||
        ch == 'i' || ch == 'I' || ch =='o' || ch=='O' ||
        ch == 'u' || ch == 'U')
        printf("%c is a vowel.\n", ch);
    else
        printf("%c is not a vowel.\n", ch);
    system("pause");
    return 0;
}
```

Check Vowels (cont'd)



- **Version 2**

```
switch (ch) {
 case 'a': case 'A':
 case 'e': case 'E':
 case 'i':  case 'I':
 case 'o': case 'O':
 case 'u': case 'U':
     printf("%c is a vowel.\n", ch);
     break;
 default:
     printf("%c is not a vowel.\n", ch);
 }
```

Check Vowels (cont'd)

- **Version 3**

```
if (check_vowel(ch))
    printf("%c is a vowel.\n", ch);
else
    printf("%c is not a vowel.\n", ch);

int check_vowel(char ch) {
  if (ch >= 'A' && ch <= 'Z')
    ch = ch - 'A' + 'a';
  if (ch == 'a' || ch == 'e' || ch == 'i' || ch == 'o' || ch == 'u')
    return 1;
  return 0;
}
```

Check Vowels (cont'd)

- **Version 4**

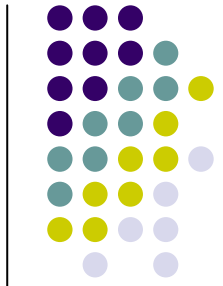
```
int check_vowel(char ch) {
    char aeiou[] = "aeiouAEIOU";
    int i;
    for (i=0; i<10; i++)
        if (ch == aeiou[i])
            return 1;
    return 0;
}
```

*(aeiou+i)

```
int check_vowel(char ch) {
    char *aeiou = "aeiouAEIOU";
    int i;
    for (i=0; i<10; i++)
        if (ch == aeiou[i])
            return 1;
    return 0;
}
```

```
int check_vowel(char ch) {
    int i;
    for (i=0; i<10; i++)
        if (ch == "aeiouAEIOU"[i])
            return 1;
    return 0;
}
```

Test multiples of 11



- 112233 is a multiple of 11 because
  -1+1-2+2-3+3 = 0, 0 is a multiple of 11
- 3234556931234861 is not a multiple of 11 because
  -3+2-3+4-5+5-6+9-3+1-2+3-4+8-6+1 = 1 is not a multiple of 11
- 9058698 is a multiple of 11, -9+0-5+8-6+9-8=-11
- Note: $a_n\ 10^n + a_{n-1}\ 10^{n-1} + a_{n-2}\ 10^{n-2} + \ldots \equiv$
  $(11\ a_n - a_n + a_{n-1})10^{n-1} + a_{n-2}\ 10^{n-2} + \ldots \equiv$
  $(11(-a_n + a_{n-1}) + a_n - a_{n-1} + a_{n-2})\ 10^{n-2} + \ldots \equiv$
  $a_n - a_{n-1} + a_{n-2} - a_{n-3} + \ldots + (-)\ a_0 \pmod{11}$
- Please write a loop to input the number character by character
  Note: you should use scanf("%c", …) instead of scanf("%d", …)
- You should process each character on the fly w/o using arrays

Approximating π

- The value for π can be determined by the series equation

  $\pi = 4 \times (1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + 1/13 - \ldots)$

- Write a program to approximate the value of π using the given formula including terms up through 1/99
- Write a program to approximate the value of π using the given formula until the variation caused by the last term is less than 0.1% of the approximated π value

Calculating x^n (mod p)

- Let x, p be two integers and n be a big integer, calculate x^n mod p (e.g. $12^{1000000000}$ mod 123457) efficiently
- If we write $n = b_k \cdot 2^k + b_{k-1} \cdot 2^{k-1} + \ldots + b_0$,
  then $x^n = x^{(((\ldots(b_k \cdot 2 + b_{k-1}) \cdot 2 + \ldots + b_3) \cdot 2 + b_2) \cdot 2 + b_1) \cdot 2 + b_0}$
  $= (((\cdots((x^{b_k})^2 x^{b_{k-1}})^2 \ldots x^{b_3})^2 x^{b_2})^2 x^{b_1})^2 x^{b_0}$

  or $x^n = x^{b_0 + 2(b_1 + 2(b_2 + 2(b_3 + \ldots + 2(b_{k-1} + 2b_k)\ldots)))}$
  $= \mathbf{x^{b_0} \cdot (x^{2^1})^{b_1} \cdot (x^{2^2})^{b_2} \cdot (x^{2^3})^{b_3} \ldots (x^{2^{k-1}})^{b_{k-1}} \cdot (x^{2^k})^{b_k}}$

- 1st step: derive $b_0, b_1, \ldots, b_k$ from n

```
q = n;
while (q>0) {
    b = q % 2;
    q = q / 2;
}
```

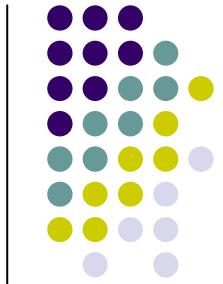
- 2nd step: x^{2^k}

```
q = n; xp = x;
while (q>0) {
    b = q % 2;
    q = q / 2;
    xp = (xp*xp)%p;
}
```

- 3rd step: $(x^{2^k})^{b_k}$

```
q = n; xp = x; rs = 1;
while (q>0) {
    if (q % 2)
        rs = (rs*xp)%p;
    q = q / 2;
    xp = (xp*xp)%p;
}
```

Calculate C_k^n and P_k^n



- **Version 1**

$\frac{n!}{k!\,(n-k)!}$

```
01 #include <stdio.h>
02 #include <stdlib.h>
03 int factorial(int);
04 int find_nCk(int, int);
05 int find_nPk(int, int);
06 int main(void) {
07     int n, k, nCk, nPk;
08     printf("Enter the value of n and k\n");
09     scanf("%d%d",&n,&k);
10     nCk = find_nCk(n, k);
11     nPk = find_nPk(n, k);
12     printf("%dC%d = %d\n", n, k, nCk);
13     printf("%dP%d = %d\n", n, k, nPk);
14     system("pause");
15     return 0;
16 }
```

```
17 int find_nCk(int n, int k) {
18     return factorial(n) / (factorial(k)*factorial(n-k));
19 }
```

$\frac{n!}{(n-k)!}$

```
20 int find_nPk(int n, int k) {
21     return factorial(n)/factorial(n-k);
22 }
```

```
23 int factorial(int n) {
24     int i, result=1;
25     for (i=1; i<=n; i++)
26         result = result*i;
27     return result;
28 }
```

Conceptually correct, actually has big problem!!

$P_3^{15} = 15 \cdot 14 \cdot 13 = 2730$

$15! = 1307674368000$

$2^{31}-1 = 2147473647$

Calculate C_k^n and P_k^n

- **Version 2 long long int**

```
01 #include <stdio.h>
02 #include <stdlib.h>
03 long long int factorial(int);
04 int find_nCk(int, int);
05 int find_nPk(int, int);
06 int main(void) {
07     int n, k, nCk, nPk;
08     printf("Enter the value of n and k\n");
09     scanf("%d%d",&n,&k);
10     nCk = find_nCk(n, k);
11     nPk = find_nPk(n, k);
12     printf("%dC%d = %d\n", n, k, nCk);
13     printf("%dP%d = %d\n", n, k, nPk);
14     system("pause");
15     return 0;
16 }
```

```
17 int find_nCk(int n, int k) {
18     return factorial(n)/(factorial(k)*factorial(n-k));
19 }
```

```
20 int find_nPk(int n, int k) {
21     return factorial(n)/factorial(n-k);
22 }
```

```
23 long long int factorial(int n) {
24     int i; long long int result=1;
25     for (i=1; i<=n; i++)
26         result = result*i;
27     return result;
28 }
```

$P_3^{21} = 21 \cdot 20 \cdot 19 = 7980$

Better! But still quite limited.

$21! = 51090942171709440000$

$2^{63}-1 = 9223372036854775807$

Calculate C_k^n and P_k^n

- **Version 3**

$$P_k^n = n\,(n-1)\cdots(n-k+1)$$

```
int find_nPk(int n, int k) {
    int i, result=1;
    for (i=0; i<k; i++)
        result = result * (n-i);
    return result;
}
```

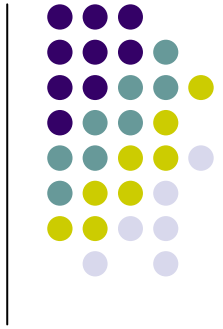
Ex. 30P6 = 427518000
30P7 = 1670497408
30P8 = -233265280
100P5 = 444567808
100P6 = -715731200

```
long long int find_nPk(int n, int k) {
    int i;
    long long int result=1;
    for (i=0; i<k; i++)
        result = result * (n-i);
    return result;
}
```

Ex. 30P13 = 7457470769548800000
30P14 = -576904376547659161 6
100P10 = 747541873440081715 2
100P11 = 8704899442529685504
100P12 = -27200710659158016

printf("result = %I64d\n", result);

Calculate C_k^n and P_k^n



- **Version 3**

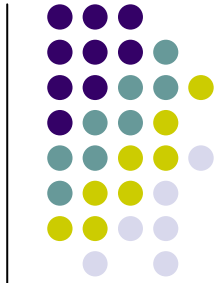
```
long long find_nCk(int n, int k)  {
    int i;
    long long int numerator=1, denominator=1;
    for (i=0; i<k; i++) {
        numerator *= n-i;
        denominator *= k-i;
    }
    return numerator/denominator;
}
```

$$C_k^n = \frac{n\ (n-1)\ \cdots\ (n-k+1)}{k\ (k-1)\ \cdots\ 1}$$

```
 long long int find_nCk(int n, int k) {
     int i;
     double result=1.0;
     for (i=0; i<k; i++)
         result *= ((double)(n-i))/(k-i);
     return (long long int) (result+0.5);
 }
```

```
long long find_nCk(int n, int k) {
    int i;
    long long numerator=1;
    long long denominator=1;
    for (i=0; i<k; i++) {
        numerator *= n-i;
        if (numerator % (k-i) == 0)
            numerator /= k-i;
        else
            denominator *= k-i;
    }
    return numerator/denominator;
}
```

Calculating e^x

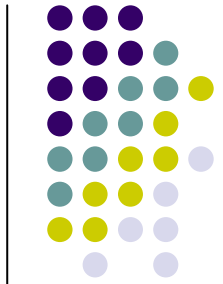


- Approximate e^x with the formula

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots \text{ until } \frac{x^n}{n!} < 10^{-5}$$

- Do not calculate factorial n! or x^n alone again!
- This is a number approximately 2.71828^x, which is larger than 2^x. If x is large, it will be a huge number. However, when you calculate n! alone, either it will overflow if you use *int* type; or the representation error is high if you use *double* type.
- It should be quite natural to use *double* type in the calculation of each term of the series.

Series approximation of e^x



```
01 #include <stdio.h>
02
03 void main()
04 {
05     double x, sum, current;
06     int i;
07     printf("Please input an exponent: ");
08     scanf("%lf", &x);
09     sum = 1 + x; i = 2; current = x * x / i;
10     while (current/sum > 0.0001)
11     {
12         sum += current;
13         i++;
14         current = current * x / i;
15     }
16     printf("e^%lf=%lf\n", x, sum);
17 }
```

Plot Sine Function in Text Mode

```
01 #include <stdio.h>
02 #include <math.h>
03 #include <stdlib.h>
04
05 void printStar(int position);
06
07 int main(void)  {
08     int i;
09     int iy;
10     const double pi = atan(1.0)*4;
11     for (i=0; i<30; i++) {
12         iy = (sin(i/30.0*2*pi)+1) * (30-1e-10) + 1; // [1.0, 60.9999999999]
13         printStar(iy);
14     }
15     system("pause");
16     return 0;
17 }
```

```
18 void printStar(int position) {
19     int i;
20     for (i=0; i<position-1; i++)
21         printf(" ");
22     printf("*\n");
23 }
```

Randomness for Simulation

- Uniform: a discrete random variable in {0, 1}, Pr[0]=Pr[1]=.5

```
#include <time.h>
#include <stdlib.h>
…
srand(time(0L));
…
for (i=0; i<100; i++) /* repeat 100 random experiments */
    printf("%d", rand()%2);
```

- Uniform: a continuous random variable in [0, 1) with probability density function f(x) = 1

```
#include <time.h>
#include <stdlib.h>
…
srand(time(0L));
…
for (i=0; i<100; i++)
    printf("%f", ((double)rand()) / RAND_MAX);
```

Specified Probability Mass/Distribution Function

- Biased coin: Pr[head]=0.3 Pr[tail]=0.7

```
#include <time.h>
#include <stdlib.h>
…
srand(time(0L));
…
for (i=0; i<100; i++) /* head: 1, tail: 0 */
    printf("%d", ((double)rand())/RAND_MAX >= 0.7 ? 1 : 0);
```

- Cheating dice: {Pr[X=1],…,Pr[X=6]} = {1/4,1/4,1/6,1/6,1/12,1/12}

```
for (i=0; i<100; i++) { /* cdf boundaries: 1/4, 1/2, 2/3, 5/6, 11/12, 1 */
    x = ((double)rand()) / RAND_MAX;
    if (x<0.25) printf("1 ");
    else if (x<0.5) printf("2 ");
    else if (x<2.0/3) printf("3 ");
    else if (x<5.0/6) printf("4 ");
    else if (x<11.0/12) printf("5 ");
    else printf("6 ");
}
```