作業二

繳交日期 112/03/16 (四) 15:10 請上傳 tronclass

1. (a) Calculate by hand 12^{-1} (mod 1729) (b) Calculate by hand the solution of equation 12 x ≡ 1124 (mod 1729). (Please write out the process of calculation.)

2. The Fibonacci numbers are defined by $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$. (a) Show that if the quotients q_i (let *a*≥*b*, $a = q_0 b + r_0$, $b = q_1 r_0 + r_1$, $r_0 = q_2 r_1 + r_2, \ldots$) appearing in the Euclidean algorithm for finding out gcd(*a*, *b*) are all equal to one then a and b are consecutive Fibonacci numbers, (b) Show that the worst case complexity of the Euclidean algorithm for finding gcd(*a*, *b*), *a*≥*b*, is O($\log_{10} b$) integer divisions. (Asymptotically, integer division has the same complexity as integer multiplication, i.e. O($\log^2$n). Thus, the complexity of Euclidean algorithm is close to an exponentiation.)

3. Solve by hand the *x*'s that satisfy the following system of congruence equations: (Please write out the process of calculation.)

$$\begin{cases} 7x \equiv 4 \pmod{93} \\ 15x \equiv 24 \pmod{39} \end{cases}$$